# MATH 147 QUIZ 11 SOLUTIONS

1. Calculate $\iint_S \sqrt{x^2+y^2+1}\, dS$ where $S$ is the helicoid given parametrically by $G(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$, with $0 \le r \le 1$ and $0 \le \theta \le 2\pi$. What is the surface area of $S$? (5pts)

   As this is a surface integral, we begin with finding the magnitude of the normal to the surface. Note $\mathbf{T}_r = (\cos\theta, \sin\theta, 0)$, $\mathbf{T}_\theta = (-r\sin\theta, r\cos\theta, 1)$, so

$$\mathbf{T}_r \times \mathbf{T}_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 1 \end{vmatrix} = (\sin\theta, -\cos\theta, r).$$

   Lastly, note that $\|\mathbf{T}_r \times \mathbf{T}_\theta\| = \sqrt{1+r^2}$. We proceed to the integral calculation itself. One has

$$\iint_s \sqrt{x^2+y^2+1}\, dS = \int_0^{2\pi}\int_0^1 (\sqrt{r^2\cos^2\theta + r^2\sin^2\theta + 1})\|\mathbf{T}_r \times \mathbf{T}_\theta\|\, dr\, d\theta = \int_0^{2\pi}\int_0^1 \sqrt{r^2+1}\sqrt{r^2+1}\, dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^1 r^2+1\, dr\, d\theta = \int_0^{2\pi} 4/3\, d\theta = 8\pi/3.$$

   To finish up the problem, note that the surface area of the helicoid is

$$\iint_S dS = \int_0^{2\pi}\int_0^1 \sqrt{r^2+1}\, dr\, d\theta.$$

2. Suppose $\mathbf{F} = x\vec{i} + y\vec{j} + (z-2)\vec{k}$. Calculate $\iint_S \mathbf{F}\cdot\, d\mathbf{S}$, for $S$ the helicoid with parameterization $G(u,v) = (u\cos(v), u\sin(v), v)$, with $0 \le u \le 1$ and $0 \le v \le 2\pi$. (5pts)

   The parameterization is the same as the first problem, so we reuse the fact that $\mathbf{T}_u \times \mathbf{T}_v = (\sin v, -\cos v, u)$. Doing so allows us to calculate

$$\iint_S F\cdot dS = \int_0^{2\pi}\int_0^1 \mathbf{F}(G(u,v))\cdot \mathbf{T}_u \times \mathbf{T}_v\, du\, dv = \int_0^{2\pi}\int_0^1 (u\cos v, u\sin v, v-2)\cdot(\sin v, -\cos v, u)\, dudv$$
$$= \int_0^{2\pi}\int_0^1 uv - 2u\, du\, dv = \int_0^{2\pi} u^2v/2 - u^2\Big|_0^1\, dv = \int_0^{2\pi} v/2 - 1\, dv = v^2/4 - v\Big|_0^{2\pi} = \pi^2 - 2\pi.$$